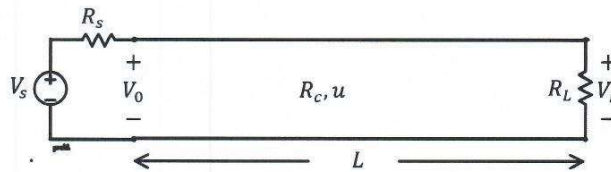


ELEC 353 – Final Exam 2019 – Solution

Question 1



1. A transmission line is $L = 14$ cm long. It has speed-of-propagation $u = 17$ cm/ns, and characteristic resistance $R_c = 73$ ohms. The load resistor is $R_L = 200$ ohms. The voltage generator V_s is a pulse function from 0 volts to 10 volts starting at $t=0$ and ending at $t=1$ ns. The internal resistance of the generator is $R_s = 100$ ohms.

1.1) What is the voltage V_0 at the generator terminals at $t=0.5$ ns?

7.14 volts	8.89	6.67	4.22	None of these
------------	------	------	------	---------------

1.2) What is the voltage V_L at the load terminals at $t=1$ ns?

6.18 volts	13.6	16.9	11.6	None of these
------------	------	------	------	---------------

1.3) What is the voltage V_0 at the generator terminals at $t=1.8$ ns?

3.29 volts	2.27	3.69	1.79	None of these
------------	------	------	------	---------------

1.4) What is the voltage V_L at the load terminals at $t=2$ ns?

1.96 volts	8.04	0.00	6.46	None of these
------------	------	------	------	---------------

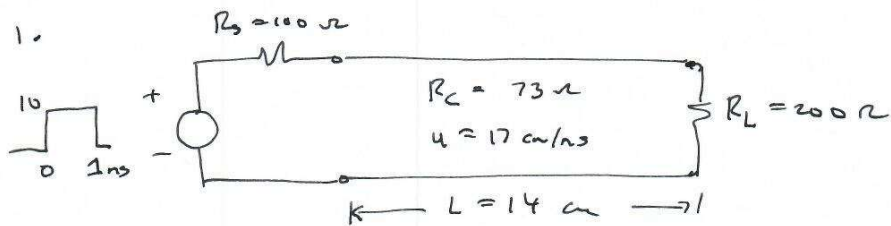
1.5) What is the voltage V_L at the load terminals at $t=2.6$ ns?

-11.9 volts	-2.86	0.449	-5.28	None of these
-------------	-------	-------	-------	---------------

1.6) What is the voltage V_0 at the generator terminals at $t=3.4$ ns?

-0.809 volts	-1.43	0.165	-1.26	None of these
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ELEC 353 Final Exam Solution 2019

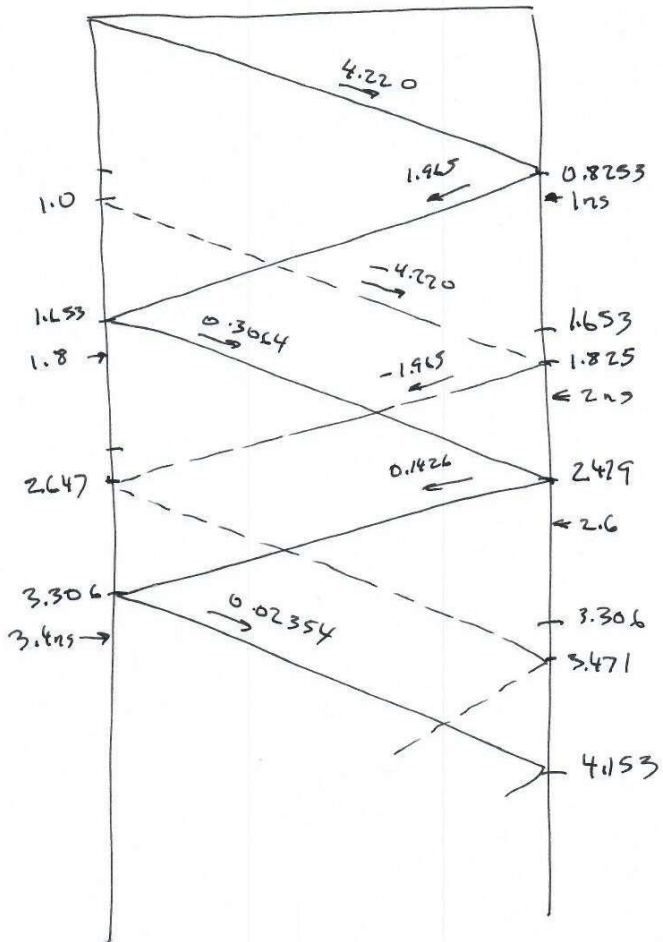


$$T = \frac{L}{u} = \frac{14}{17} = 0.8235 \text{ ns}$$

$$V^+ = \frac{R_c V_s}{R_s + R_c} = \frac{73 \times 10}{73 + 100} = 4.220 \text{ V.}$$

$$\Gamma_s = \frac{R_s - R_c}{R_s + R_c} = \frac{100 - 73}{100 + 73} = 0.1561$$

$$\Gamma_L = \frac{R_L - R_c}{R_L + R_c} = \frac{200 - 73}{200 + 73} = 0.4652$$



1.1) V_0 at 0.5 ns

$$V_0 = 4.220$$

1.2) V_L at 1 ns

$$V_L = 4.220 + 1.965 \\ = 6.185$$

1.3) V_0 at 1.8 ns

$$V_0 = 4.220 + 1.965 \\ + 0.3064 - 4.220 \\ = 6.491 - 4.220 \\ = 2.271$$

1.4) V_L at 2 ns

$$V_L = 4.220 + 1.965 \\ - 4.220 - 1.965 \\ = 0$$

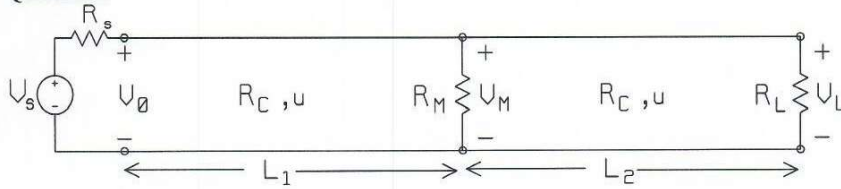
1.5) V_L at 2.6 ns

$$V_L = 0 + 0.3064 + 0.1426 \\ = 0.4490$$

1.6) V_0 at 3.4 ns

$$V_0 = 0.1426 + 0.62354 \\ = 0.7661$$

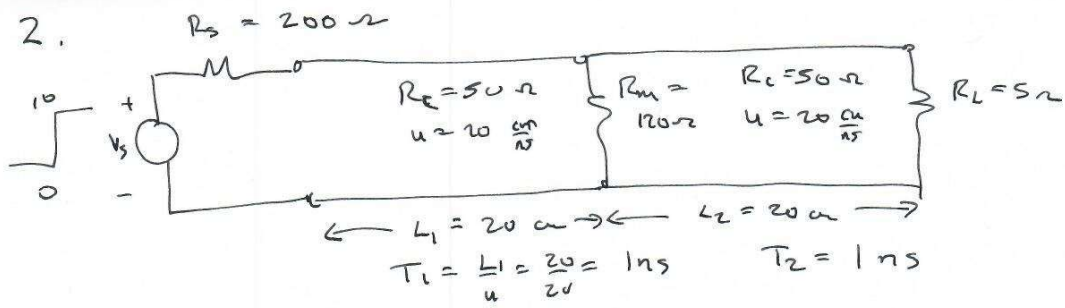
Question 2



The transmission-line circuit shown above has a step function generator starting at $t = 0$ of height $V_s = 10$ volts and internal resistance $R_s = 200$ ohms. The two transmission lines are identical, with characteristic

resistance $R_C = 50$ ohms and speed of travel $u = 20$ cm/ns. The lengths are $L_1 = L_2 = 20$ cm. The load resistors are $R_M = 120$ ohms and $R_L = 5$ ohms.

2.1 What is the voltage V_0 across the generator terminals at $t = 0.4$ ns?	6.66 volts	3.33	9.80	2.00	None of these
2.2 What is transmission coefficient at the junction?	0.29	0.67	0.83	0.76	None of these
2.3 What is the voltage V_M across the junction at $t = 1.2$ ns?	2.80 volts	4.44	2.54	1.66	None of these
2.4 What is the voltage V_L across the load at $t = 2.4$ ns?	1.69 volts	4.48	5.93	0.30	None of these
2.5 What is the voltage V_0 across the generator at $t = 2.4$ ns?	1.45 volts	9.53	2.28	5.20	None of these
2.6 What is the voltage V_M at the junction at $t = 3.2$ ns?	5.20 volts	5.93	0.363	1.69	None of these
2.7 What is the final value of the load voltage V_L as $t \rightarrow \infty$?	0.23 volts	9.05	1.60	5.71	None of these



$$V^+ = \frac{R_c V_s}{R_s + R_c} = \frac{50 \times 10}{200 + 50} = 2 \text{ volts}$$

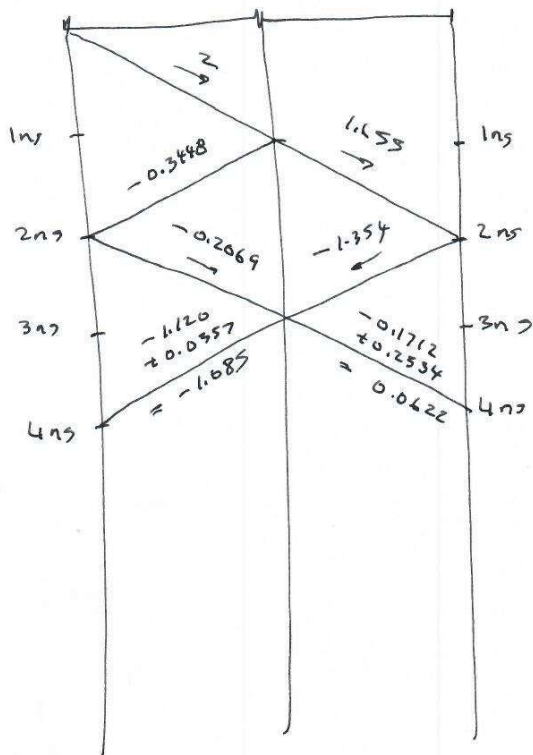
$$\Gamma_s = \frac{R_s - R_c}{R_s + R_c} = \frac{200 - 50}{200 + 50} = \frac{150}{250} = 0.6$$

$$\Gamma_L = \frac{R_L - R_c}{R_L + R_c} = \frac{5 - 50}{5 + 50} = -0.8182$$

$$R_p = R_m \parallel R_c = \frac{120 \times 50}{120 + 50} = 35.29$$

$$\Gamma_J = \frac{R_p - R_c}{R_p + R_c} = \frac{35.29 - 50}{35.29 + 50} = -0.1724$$

$$T_J = \frac{2 R_p}{R_p + R_c} = \frac{2 \times 35.29}{35.29 + 50} = 0.8275$$



2.1) V_0 at 0.4 ns

$$V_0 = 2 \text{ v. at } t_0$$

2.2) $T_J = 0.8275$

2.3) V_m at 1.2 ns

$$V_m = 2 - 0.3448 = 1.655$$

2.4) V_L at 2.4 ns

$$V_L = 1.655 - 1.354 = 0.3012$$

2.5) V_0 at 2.4 ns

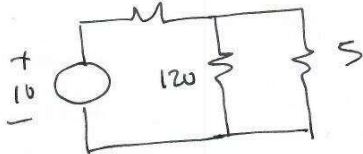
$$V_0 = 2 - 0.3448 - 0.2069 = 1.448 \text{ v.}$$

2.6) $V_m = 1.655 - 0.2069$

$$= 1.085 = 0.3631$$

2.7) FINAL VALUE

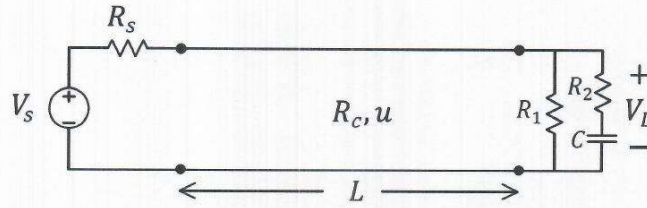
200Ω



$$120 \parallel 120 = 4.8 \Omega$$

$$V_L = 4.8 \times \frac{10}{200 + 4.8} = 0.2344$$

Question 3



A transmission line has length $L = 980$ cm, characteristic resistance $R_c = 300$ ohms and speed of travel $u = 17$ cm/ns. The generator is a step function starting at $t = 0$, of amplitude $V_s = 10$ volts and internal resistance $R_s = 300$ ohms. The line is terminated by a load as shown in the figure, with two resistors $R_1 = 120$ ohms and $R_2 = 20$ ohms, and a capacitor $C = 0.1636$ pF. The capacitor is initially uncharged.

3.1 At what time does the exponential charging start?

57.7 ns	42.4	30.0	19.4	None of these
---------	------	------	------	---------------

3.2 What is the time constant of the exponential charging?

9.00 ns	5.82	17.3	12.7	None of these
---------	------	------	------	---------------

3.3 What is the initial value of the load voltage V_L when the exponential charging starts?

1.42 volts	0.54	1.82	1.12	None of these
------------	------	------	------	---------------

3.4 What is the final value of the exponential charging of the load voltage V_L ?

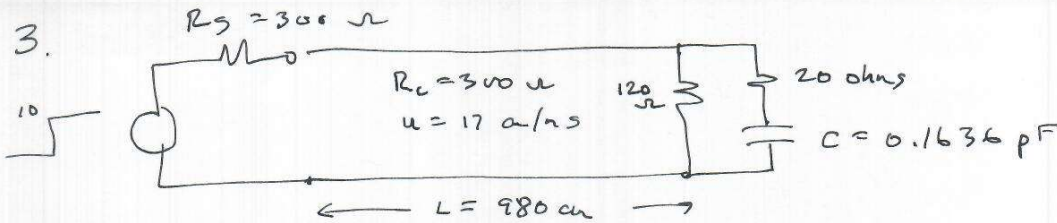
3.41 volts	2.86	5.44	2.24	None of these
------------	------	------	------	---------------

3.5 Write the formula for the voltage across the capacitor. (2 marks)

$$V_L(t) = 2.857 - 2.317 e^{-\frac{t - 57.65}{17.3 \times 10^{-9}}} \quad (t \text{ in ns})$$

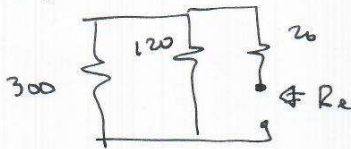
3.6 What is the value of the voltage V_L at $t = 107.8$ ns?

2.38 volts	4.43	2.73	1.83	None of these
------------	------	------	------	---------------



3.1) $T = \frac{L}{u} = \frac{980}{17} = 57.65 \text{ ns}$

3.2)



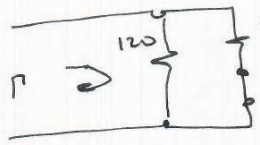
$$300 \parallel 120 = 85.714$$

$$R_e = 20 + 85.714 = 105.714$$

$$T = R_e C = 105.714 \times 0.1636 \times 10^{-12} = 17.295 \text{ ps}$$

Since the units of the answers on the exam paper were "ns" the correct answer is "none of these".

3.4) Initial value: C is a short circuit.



$$R_L = 120 \parallel 20 = 17.14$$

$$\Gamma = \frac{R_L - R_c}{R_L + R_c} = \frac{17.14 - 300}{17.14 + 300}$$

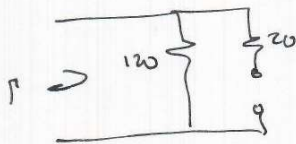
$$= -0.8919$$

$$V^+ = 5$$

$$V^- = \Gamma V^+ = 5 \times (-0.8919) = -4.460$$

$$V_L = V^+ + V^- = 5 - 4.460 = 0.540$$

3.5) Final Value: C is an open circuit



$$\Gamma = \frac{120 - 300}{120 + 300} = -0.4286$$

$$V^+ = 5$$

$$V^- = 5 \Gamma = -2.143$$

$$V_L = V^+ + V^- = 5 - 2.143$$

$$= 2.857$$

$$3.6) V_L(t) = V_{\text{final}} + (V_{\text{initial}} - V_{\text{final}}) e^{-\frac{t-T}{\tau}}$$

$$= 2.857 + (0.5400 - 2.857) e^{-\frac{t - 57.65}{17.295 \times 10^{-3}}}$$

with t in ns.

$$V_L(t) = 2.857 - 2.317 e^{-\frac{t - 57.65}{17.295 \times 10^{-3}}}$$

$$3.7) t = 107.8 \text{ ns}$$

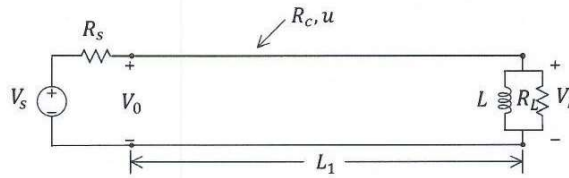
$$V_L(107.8) = 2.857 - 2.317 e^{-\frac{107.8 - 57.65}{17.295 \times 10^{-3}}}$$

$$= 2.857 - 2.317 e^{-2.900}$$

$$= 2.857 \text{ volts}$$

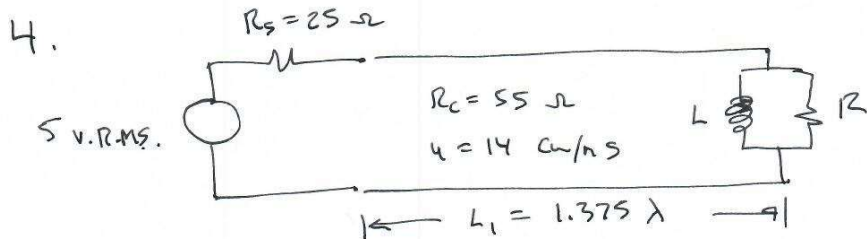
The correct answer is "none of these".

Question 4



A transmission line circuit operates at a frequency of 2450 MHz. The transmission line has characteristic resistance $R_c = 55$ ohms and speed of travel $u = 14$ cm/ns. The length of the transmission line is $L_1 = 1.375\lambda$ where λ is the wavelength on the transmission line. The load impedance consists of an inductance of $L = 14$ nanoHenries in parallel with a resistance $R_L = 30$ ohms. The A.C. generator has R.M.S. value $V_s = 5$ volts. The internal resistance is $R_s = 25$ ohms.

- 4.1 Find the impedance of the inductance in parallel with the load resistance at the operating frequency.
- | | | | | |
|-------------------|--------------|-------------|--------------|---------------|
| 55.68+j15.50 ohms | 67.37+j29.17 | 29.43+j4.10 | 70.32+j45.68 | None of these |
|-------------------|--------------|-------------|--------------|---------------|
- 4.2 What is the length of the transmission line in cm?
- | | | | | |
|---------|------|------|------|---------------|
| 7.86 cm | 6.43 | 5.00 | 3.57 | None of these |
|---------|------|------|------|---------------|
- 4.3 What is the magnitude and angle of the reflection coefficient at the load?
- | | | | | |
|-------------|-------------|--------------|-------------|---------------|
| 0.139∠79.5° | 0.252∠53.6° | 0.306∠168.1° | 0.361∠51.4° | None of these |
|-------------|-------------|--------------|-------------|---------------|
- 4.4 What is the standing-wave ratio (SWR) on the transmission line?
- | | | | | |
|------|------|------|------|---------------|
| 2.13 | 1.67 | 1.89 | 1.32 | None of these |
|------|------|------|------|---------------|
- 4.5 Find the input impedance of the transmission line circuit.
- | | | | | |
|-------------------|-------------|--------------|--------------|---------------|
| 78.30-j24.98 ohms | 41.74+j2.15 | 84.55-j43.80 | 40.85-j27.03 | None of these |
|-------------------|-------------|--------------|--------------|---------------|
- 4.6 Find the R.M.S. value of the voltage V_0 at the input to the transmission line.
- | | | | | |
|------------|------|------|------|---------------|
| 4.04 volts | 3.44 | 3.87 | 3.13 | None of these |
|------------|------|------|------|---------------|



$$f = 2450 \text{ MHz} \quad \omega = 2\pi f = 1.539 \times 10^{10}$$

$$L = 14 \text{ nH}$$

$$R_L = 30 \text{ ohms}$$

4.1) $Z_L = j\omega L = j 1.539 \times 10^{10} \times 14 \times 10^{-9} = j215.5 \text{ ohms}$

$Z_{\text{load}} = 30 \parallel j215.5 = 29.43 + j4.096 \text{ ohms}$

4.2) $\lambda = \frac{u}{f} = \frac{14}{2.450} = 5.714 \text{ cm}$

$L = 1.375 \lambda = 7.857 \text{ cm}$

$$4.3) \Gamma_L = \frac{Z_{L_{\text{max}}} - R_c}{Z_{L_{\text{max}}} + R_c} = \frac{(29.43 + j4.096) - 55}{(29.43 + j4.096) + 55}$$

$$= -0.29979 + j0.04305$$

$$= 0.3063 \angle 168.1^\circ$$

$$4.4) \text{SWR} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = 1.883$$

$$4.5) \beta = \frac{2\pi}{\lambda} = \frac{360}{0.05714} = 6300 \text{ deg/m}$$

$$\beta L = \frac{2\pi}{\lambda} 1.375 \lambda = 360 \times 1.375 = 495^\circ$$

$$\tan \beta L = -1$$

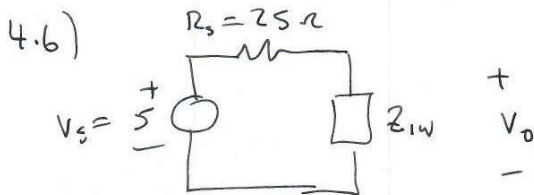
$$Z_{in} = R_c = \frac{Z_{L_{\text{max}}} + jR_c \tan \beta L}{R_c + jZ_{L_{\text{max}}} \tan \beta L}$$

$$Z_{in} = 55 \frac{29.43 + j4.096 + j55(-1)}{55 + j(29.43 + j4.096)(-1)}$$

$$= 55 \frac{58.80 \angle -59.77^\circ}{66.02 \angle -26.47^\circ}$$

$$= 48.985 \angle -33.5^\circ$$

$$= 40.85 - j27.04$$



$$V_o = Z_{in} \frac{V_s}{Z_{in} + R_s} = \frac{48.985 \angle -33.5^\circ \times 5}{48.985 \angle -33.5^\circ + 25}$$

$$= 3.441 \angle -11.18^\circ$$

V_o RMS is 3.441 V.

Question 5

A plane wave travels in the z direction in a material at 1500 MHz. The relative permittivity of the material is $\epsilon_r = 51$ and the loss tangent is 0.05. The electric field is oriented parallel to the x axis. The amplitude of the electric field at $z=0$ is 5 volts/meter and the phase is zero degrees. The material is non-magnetic.

5.1 What is the value of the conductivity?

63.8 mS/m	8.68	267	213	None of these
-----------	------	-----	-----	---------------

5.2 What is the phase constant?

224.6 rad/meter	71.7	14.4	53.6	None of these
-----------------	------	------	------	---------------

5.3 What is the penetration depth?

1.40 m	0.250	0.178	0.113	None of these
--------	-------	-------	-------	---------------

5.4 What is the amplitude of the electric field at $z = 37.43$ cm?

1.36 V/m	0.913	0.612	1.12	None of these
----------	-------	-------	------	---------------

5.5 What is the magnitude of the intrinsic impedance of the material?

124.9 ohms	164.8	65.1	52.7	None of these
------------	-------	------	------	---------------

5.6 What is the amplitude of the magnetic field at $z = 0$?

40.04 mA/m	30.34	76.22	94.84	None of these
------------	-------	-------	-------	---------------

5) $f = 1500 \text{ MHz}$ $\omega = 2\pi f = 9.424 \times 10^9 \text{ r/s}$
 $\epsilon_r = 51$
 $\tan \delta = 0.05$
 $E_0 = 5 \text{ V/m}$

5.1) $\tan \delta = \frac{\sigma}{\omega \epsilon}$

$$\sigma = \omega \epsilon \tan \delta = 9.424 \times 10^9 \times 51 \times 8.854 \times 10^{-12} \times 0.05$$

$$= 0.2128 = 212.8 \text{ mS/m}$$

5.2) $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

$$= \sqrt{j \cdot 9.424 \times 10^9 \times 4\pi \times 10^{-7} (0.2128 + j \cdot 9.424 \times 10^9 \times 51 \times 8.854 \times 10^{-12})}$$

$$= \sqrt{j \cdot 11,842.5 (0.2128 + j \cdot 4.2554)}$$

$$= \sqrt{11,842.5 \angle 90 (4.261 \angle 87.13)}$$

$$= \sqrt{50,461 \angle 177.13}$$

$$= 224.635 \angle 88.565$$

$$= 5.651 + j 225.56$$

$\beta = 225.56 \text{ r/m}$

$$5.3) \delta = \frac{1}{\alpha} = \frac{1}{5.65} = 0.17699 = 17.7 \text{ cm}$$

$$5.4) E(37.43 \text{ cm}) = 5e^{-\alpha z} = 5e^{-5.65 \times 0.3743}$$

$$= 5 \times 0.11207$$

$$= 0.6033 \text{ V/m}$$

$$5.5) \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{11,842.5 \text{ k}\Omega}{4.261487113}}$$

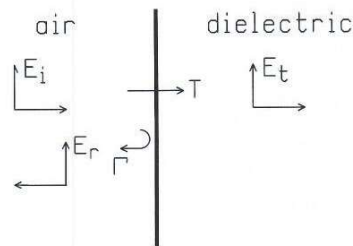
$$= \sqrt{2879.4287} = 52.72 \angle 1.44^\circ$$

$|\eta| = 52.72$

$$5.6) |H| = \frac{|E|}{|\eta|} = \frac{5}{52.72} = 0.09484 \text{ A/m}$$

$$= 94.84 \text{ mA/m}$$

Question 6



6. A plane wave in air at 3.20 GHz has amplitude $E_i = 12.0 \text{ mV/m}$ and travels in the $+z$ direction. The xy plane is the interface between air for $z < 0$ and plastic for $z > 0$. The plastic material has $\epsilon_r = 7.1$ and is lossless and non-magnetic. The observer is in the air at $z = -1.56 \text{ cm}$.

6.1) What is the intrinsic impedance of the plastic material?

104 ohms	248	281	141	none of these
----------	-----	-----	-----	---------------

6.2) What is the reflection coefficient at the surface of the plastic?

-0.567	-0.205	-0.146	-0.454	none of these
--------	--------	--------	--------	---------------

6.3) What is the amplitude and phase of the incident electric field strength at the observer?

8.00 mV/m $\angle -135^\circ$	42.5 mV/m $\angle 135^\circ$	12.0 mV/m $\angle 60^\circ$	14.0 mV/m $\angle -45^\circ$	none of these
-------------------------------	------------------------------	-----------------------------	------------------------------	---------------

6.4) What is the amplitude and phase of the reflected electric field strength at the observer?

2.04 mV/m $\angle -135^\circ$	5.45 mV/m $\angle 120^\circ$	8.72 mV/m $\angle 45^\circ$	4.54 mV/m $\angle -45^\circ$	none of these
-------------------------------	------------------------------	-----------------------------	------------------------------	---------------

6.5) What is the amplitude of the electric field strength at the observer?

9.20 mV/m	43.4	15.5	14.1	none of these
-----------	------	------	------	---------------

6.6) What is the power density transmitted into the plastic?

57.6 nanoW/m ²	255	152	2230	none of these
---------------------------	-----	-----	------	---------------

$$6) \quad \begin{array}{l|l} \text{air} & \text{dielectric} \\ \hline \uparrow E_i = 12.0 & \epsilon_r = 7.1 \\ \text{V/m} & \\ \\ x & \\ z = -1.56 \text{ m} & \end{array}$$

$$f = 3.20 \text{ GHz}$$

$$6.1) \quad \eta_2 = \frac{\eta_0}{\sqrt{\epsilon_r}} = \frac{377}{\sqrt{7.1}} = 141.5 \, \Omega$$

$$6.2) \quad \Gamma = \frac{\eta_2 - \eta_0}{\eta_2 + \eta_0} = \frac{141.5 - 377}{141.5 + 377} = -0.4542$$

$$6.3) \quad E_i = 12 e^{-j\beta z}$$

$$\lambda = \frac{v}{f} = \frac{3 \times 10^8}{3.20 \times 10^9} = 0.09375 \text{ m}$$

$$\beta = \frac{360}{\lambda} = 3.840 \text{ deg/m}$$

$$\beta z = 3.840 \times (-0.0156) = -59.9^\circ$$

$$E_i = 12 e^{-j\beta z} = 12 e^{-j(-59.9^\circ)} = 12 \angle 59.9^\circ$$

$$6.4) \quad \begin{aligned} E_r &= \Gamma E_i e^{+j\beta z} = -0.4542 \times 12 \times e^{-j59.9^\circ} \\ &= -5.45 \times e^{-j59.9^\circ} = 5.45 e^{j180^\circ} e^{-j59.9^\circ} \\ &= 5.45 e^{j120.1^\circ} = 5.45 \angle 120.1^\circ \end{aligned}$$

$$6.5) \quad E = E_i + E_r = 12 \angle 59.9^\circ + 5.45 \angle 120.1^\circ$$

$$= 3.285 \angle +j 15.097$$

$$= 1.545 \angle 77.7^\circ$$

$$|E| = 1.545$$

$$6.6) E_t = T E_i$$

$$T = \frac{2\eta_c}{\eta_c + \eta_0} = \frac{2 \times 141.5}{141.5 + 377} = 0.5458$$

$$E_t = 0.5458 \times 12 = 6.55 \text{ mV/m}$$

$$S_{av} = \frac{|E_t|^2}{2\eta} = \frac{(0.00655)^2}{2 \times 141.5}$$

$$= 1.516 \times 10^{-7} = 0.1516 \times 10^{-6} \text{ W/m}^2$$

$$= 151.6 \text{ nW/m}^2$$

Question 7

A wireless link is set up at 14 GHz between a transmit antenna of gain $G_T = 8$ dB, and a receive antenna of gain $G_R = 9$ dB. The antennas are 4 km apart. The input power to the transmitter is 1 kW. The antennas are lossless.

7.1 What is the spreading factor for the wireless link?

-135 dB	-127	-141	-147	None of these
---------	------	------	------	---------------

7.2 What is the power flow density S_{av} in microwatts per square meter at the location of the receiver?

31.38 microWatts/m ²	6.17	0.32	12.37	None of these
------------------------------------	------	------	-------	---------------

7.3 What is the effective area of the receive antenna?

0.000290 m ²	0.000145	0.000578	0.000365	None of these
-------------------------	----------	----------	----------	---------------

7.4 The receive antenna is terminated in a matched load. What is the power delivered to the matched load?

9.10 nW	1.80	2.25	0.18	None of these
---------	------	------	------	---------------

7.5 The system designer is considering a different antenna separation of 14 km apart. What gain G_R is needed for the receive antenna so that the received power remains the same as in question 7.4?

7.82 dB	6.89	12.02	19.88	None of these
---------	------	-------	-------	---------------

$$7 \quad f = 14 \text{ GHz}$$

$$G_T = 8 \text{ dB}$$

$$\rightarrow 10^{8/10} = 6.31$$

$$G_R = 9 \text{ dB}$$

$$\rightarrow 10^{9/10} = 7.94$$

$$R = 4 \text{ km}$$

$$P_t = 1 \text{ kW}$$

$$7.1) L_t = \left(\frac{\lambda}{4\pi R} \right)^2$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{14 \times 10^9} = 0.02143 \text{ m}$$

$$L_t = \left(\frac{0.02143}{4 \times \pi \times 4000} \right)^2$$

$$= (4.263 \times 10^{-7})^2$$

$$= 1.817 \times 10^{-13}$$

$$= -127.4 \text{ dB}$$

$$7.2) S_{av} = G_T \frac{P_t}{4\pi R^2} = 6.31 \frac{1000}{4\pi (4000)^2}$$

$$= 3.138 \times 10^{-5} = 31.38 \text{ } \mu\text{W/m}^2$$

$$7.3) A_e = \frac{\lambda^2}{4\pi} G_R = \frac{(0.02143)^2}{4\pi} \times 7.94$$

$$= 2.902 \times 10^{-4} \text{ m}^2$$

$$= 0.0002902 \text{ m}^2$$

$$7.4) P_R = A_e S_{av} = 9.106 \times 10^{-3} \text{ } \mu\text{W} = 9.106 \text{ nW}$$

$$7.5) P_R = \left(\frac{\lambda}{4\pi R} \right)^2 G_R G_T P_t$$

$$R = 14 \text{ km}$$

$$G_R = \frac{P_R}{\left(\frac{\lambda}{4\pi R} \right)^2 G_T P_t}$$

$$\left(\frac{\lambda}{4\pi R} \right)^2 = \left(\frac{0.02143}{4\pi \times 14000} \right)^2 = (1.218 \times 10^{-7})^2$$

$$= 1.484 \times 10^{-14}$$

$$G_{12} = \frac{9.102 \times 10^{-9}}{1.484 \times 10^{14} \times 6.31 \times 1000}$$
$$= 97.24 \rightarrow 19.88 \text{ dB}$$