NON-LOCAL COHERENT DENOISING OF RF DATA FOR ULTRASOUND ELASTOGRAPHY

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ABSTRACT

Ultrasound elastography infers mechanical properties of living tissues from ultrasound radio frequency (RF) data recorded while the tissues are undergoing deformation. A challenging yet critical step in ultrasound elastography is to estimate the tissue displacement (or, equivalently the time delay estimate) fields from pairs of RF data. The RF data is often corrupted with noise, which causes the displacement estimator to fail in many *in-vivo* experiments. To address this problem, we present a non-local, coherent denoising approach based on Bayesian estimation to reduce the impact of noise. Despite incoherent denoising algorithms that smooth the B-mode images, the proposed denoising algorithm is used to suppress noise while maintaining useful information such as speckle patterns. We refer to the proposed approach as COherent Denoising for Elastography (CODE) and evaluate its performance when CODE is used in conjunction with two state-of-art elastography algorithms, namely: (i) GLobal Ultrasound Elastography (GLUE) [1], and; (ii) Dynamic Programming Analytic Minimization elastography (DPAM) [2]. Our results show that CODE substantially improves the strain result of both GLUE and DPAM.

Index Terms— Ultrasound, Elastography, Coherent Denoising, Radio-Frequency, Non-local, Time Delay Estimation.

1. INTRODUCTION

Ultrasound elastography determines the viscoelastic properties of tissues, and is useful for diagnosis of pathology and for aiding surgeons in the operating room. Broadly speaking, ultrasound elastography can be grouped into two categories [3]-[8]: dynamic elastography and quasi-static elastography. In this paper, we focus on two state-of-art freehand palpation, quasi-static elastographic approaches, namely Global Time-Delay Estimation in Ultrasound Elastography (GLUE) [1] and Real-Time Regularized Ultrasound Elastography (DPAM) [2]. Both approaches use successive pairs of frames of ultrasound RF data to estimate the tissue displacement (also referred to as time delay estimates (TDE)). The derivative of TDE provides an estimate of the induced strain that represents the stiffness or softness of the tissue being im-

aged. Figure 1 illustrates the steps involved in quasi-static ultrasound elastography with the hand-held device shown on the left hand-side and the displacement field estimates defined using the two frames on the right.

At the heart of both GLUE and DPAM is an energy minimization approach to determine TDE's [1, 2] . A dynamic programming approach is used in both cases to compute TDE's first at a coarse pixel level. The resolution of the TDE's is then enhanced to the finer subpixel level through analytical minimization. Given that RF ultrasound data can be corrupted by several factors such as thermal and electronic noise, there is a need to compensate for noise in the RF data. Traditional filtering techniques, such as the convolution with a Guassian kernel, use local continuity in the images to reduce noise. A new class of denoising algorithms, referred to as non-local means (NLM) [9], considers data from a much larger "non-local" region for denoising. NLM relies on redundancy in images and uses the weighted average of most similar intra-frame pixels within a large non-local neighbourhood to eliminate noise.

Most NLM based denoising approaches [10, 11, 12] remove noise from processed output of the RF data, which is referred to as B-mode images in ultrasound literature. NLM denoising reduces speckle pattern and generates smooth B-mode images. Ultrasound speckle is useful in several image analysis techniques, such as ultrasound elastography [13, 14], freehand sensorless 3D ultrasound [15, 16] and quantitative ultrasound [17]. In this work, we focus on ultrasound elastography.

In this paper, we present an alternate approach wherein NLM denoising algorithm is applied directly to raw RF data instead of processed B-mode images. We refer to the proposed approach as COherent Denoising for Elastography (CODE) and evaluate its performance on *in-vivo* liver ablation data when used in conjunction with two commonly used elastography algorithms, namely: (i) GLobal Ultrasound Elastography (GLUE) [1], and; (ii) Dynamic Programming Analytic Minimization elastography (DPAM) [2]. CODE exploits the complete set of information in the RF domain, some of which is likely to be lost in the processing steps used to generate the B-mode images. It is, therefore, our intuition that CODE would result in superior denoising results. Using information in RF data to generate visually informative B-

mode images is challenging [18]. To illustrate the superiority of CODE, both mathematical analysis and experimental results are included in the paper. Our comparisons corroborate our intuition and verify the usefulness of CODE.

The rest of this paper is organized as follows. In Section 2, we introduce GLUE and DPAM as representative quasi-static elastography approaches. Section 3 provides background on non-local denoising and introduces CODE as a Bayesian estimator. In Section 4, we explore the ability of CODE on simulation data. Experimental results using phantom and *in-vivo* data are included in Section 5. Finally, we conclude the paper in Section 6.

2. QUASI-STATIC ELASTOGRAPHY: GLUE AND DPAM

Both DPAM and GLUE are quasi-static approaches based on the optimizing a regularized cost function to determine tissue displacements. They both aim at finding the axial and lateral displacements (a and l) of all samples of RF data as shown in Figure 1. DPAM uses dynamic programming (DP) to first estimate the integer displacement of a *seed-line* in terms of the number of pixels and then applies analytical minimization (AM) to fine tune the estimated displacement to the subpixel level. The strain image is obtained using the spatial differentiation of the displacement field. GLUE also uses DP for estimating the integer tissue displacements and refines the estimates to subpixels for the entire image simultaneously. In other words, GLUE solves an optimization function where both axial and lateral displacement of every sample of the RF frame are unknowns, i.e. in the order of a million variables. This is in contrast to DPAM, which refines the estimates line-by-line. The strain image again is calculated based on the differentiation of displacement map similar to DPAM. Although GLUE and DPAM perform well in most cases, they may not converge to the correct solution in the presence of excessive noise. In the next section, we present our denoising approach used to reduce the impact of noise in the RF domain.

3. THE NON-LOCAL DENOISING APPROACH

The central idea behind this paper is to apply coherent denoising on RF data. Unlike incoherent denoising approaches that process the B-mode images to remove noise (resulting in spatial averaging and significant loss of speckle patterns), the proposed approach retains speckle patterns. We first outline NLM, which followed by a description of the CODE algorithm, including an analytical justification of why CODE provides better denoising results.

3.1. Non-local means

Let $v(i)$ be the observed value of the discretized image for pixel i and $u(i)$ be its true value. Due to the presence of noise

$n(i)$, we have

$$v(i) = u(i) + n(i). \qquad (1)$$

To simplify our explanation, we focus on 1D signals but our results are generalizable to 2D images. In fact, the experimental results included in Section 4 are for 2D phantom and in-vivo liver ablation data. To denoise the image for each pixel i, NLM searches a reference area of the image within a rectangular *search window* Δ_i, which is centered around pixel i (see Figure 2). A neighbourhood N_i of known dimension is selected around pixel i and compared to neighbourhood N_j around pixel j for all $j \in \Delta_i$. For pixel i, weight $w(i, j)$ is assigned to each pixel j. The value of pixel i is then replaced by

$$\text{NLM}[v](i) = \sum_{j \in \Delta_i} w(i, j) * v(j). \qquad (2)$$

The distance metric is proportional to the square of Euclidian distance between the two patches. The weight is then calculated as

$$w(i, j) = \frac{1}{Z_i} \exp \left\{ -\frac{||v(N_i) - v(N_j)||_{2,a}^2}{h^2} \right\}. \qquad (3)$$

Based on (3), it is clear that the weight is the convolution of a Gaussian with standard deviation $a > 0$ and the squared Euclidean distance between two neighbourhoods $||v(N_i) - v(N_j)||_2^2$, for N_i and N_j. The smoothing parameter h controls the contribution of the Guassian-Euclidean distance exponent in the weights. The normalization factor Z_i for pixel i is given by

$$Z(i) = \sum_{j \in \Delta_i} \exp \left\{ -\frac{||v(N_i) - v(N_j)||_{2,a}^2}{h^2} \right\}, \qquad (4)$$

where the weight is normalized to ensure that the dynamic range of the $\text{NLM}[v](i)$ is the same as that of its counterpart $v(i)$.

3.2. The proposed Bayesian CODE framework

Noise in ultrasound B-mode images originates from piezo-electric sensors and data acquisition card. Depending on the application, the level of noise can even be higher. For example, ablation treatment generates heat and microbubbles that severely deteriorate RF data [19, 20, 21]. Both logarithmic compression and envelope detection steps, applied to derive the B-mode image, are non-linear operations that complicate measurement noise added by sensors and acquisition card. Our CODE approach eliminates noise introduced by sensors and acquisition card before the non-linear logarithmic compression and envelope detection by applying NLM directly to RF ultrasound data.

We now provide an analytical explanation of why NLM denoisingis is adapted for the RF domain. Let $\mathbf{g(x)}$ and $\mathbf{o(x)}$ be vectorized ground truth and observed patches of size n centered at pixel x_i of RF data (Figure 2). We define them as
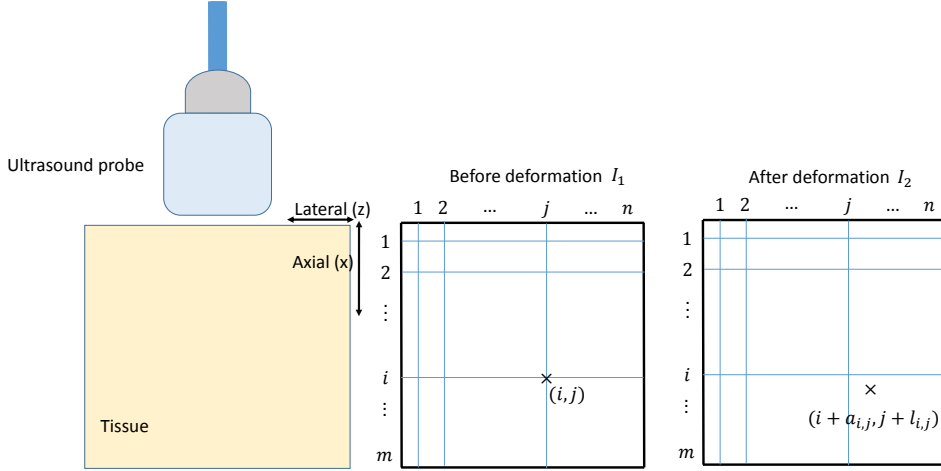
Fig. 1. Illustration of ultrasound elastography. The left figure shows a hand-held device that induces an external stimulus into the tissues. The pair of figures on the right include two successive frames I_1 and I_2 of RF data. The goal of ultrasound elastography is to find the displacement $(a_{i,j}, l_{i,j})$ for each pixel (i,j) in the I_1 RF frame.
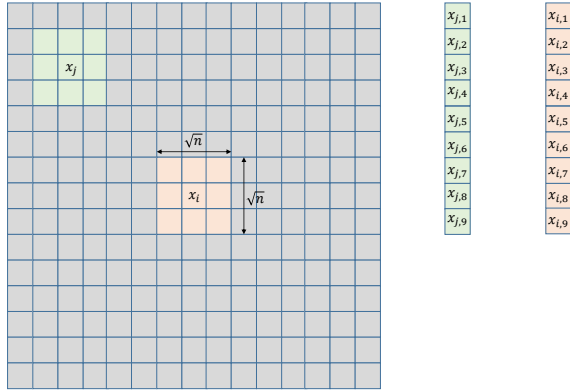
Fig. 2. Illustration for the patch and vectorized indices used in the proposed CODE approach for $n = 9$.

$\mathbf{g}(\mathbf{x}) := g(x_k)$ with $x_k \in N_g(x)$ and $\mathbf{o}(\mathbf{x}) := o(x_k)$, where $x_k \in N_o(x)$ and $\{N_o, N_g\}$ are the neighbourhoods (patches) of size $(\sqrt{n} \times \sqrt{n})$ around the central pixel x in ground truth and observed images. Our goal is to derive the Bayesian estimator $\widehat{\mathbf{g}}(\mathbf{x})$ for patch $\mathbf{g}(\mathbf{x})$ based on the observed patch $\mathbf{o}(\mathbf{x})$. Defining the optimal estimator by minimizing the posterior expected loss as

$$\mathbb{E}[L(\mathbf{g}(\mathbf{x}), \widehat{\mathbf{g}}(\mathbf{x}))] = \sum_{\mathbf{g}(\mathbf{x}) \in \Gamma} [L(\mathbf{g}(\mathbf{x}), \widehat{\mathbf{g}}(\mathbf{x}))] p(\mathbf{g}(\mathbf{x})|\mathbf{o}(\mathbf{x})), \quad (5)$$

where Γ constitutes all possible outcomes of $\mathbf{g}(\mathbf{x})$, the loss function is given by

$$L(\mathbf{g}(\mathbf{x}), \widehat{\mathbf{g}}(\mathbf{x})) = \|\mathbf{g}(\mathbf{x}) - \widehat{\mathbf{g}}(\mathbf{x})\|^2. \quad (6)$$

Substituting (6) in (5), the optimal Bayesian estimator is

$$\widehat{\mathbf{g}}(x)_{opt} = \arg \min_{\widehat{\mathbf{g}}(x)} \sum_{\mathbf{g}(\mathbf{x})} \|\mathbf{g}(\mathbf{x}) - \widehat{\mathbf{g}}(x)\|^2 \, p(\mathbf{g}(\mathbf{x})|\mathbf{o}(\mathbf{x}))$$

$$= \sum_{\mathbf{g}(\mathbf{x})} \mathbf{g}(\mathbf{x}) p(\mathbf{g}(\mathbf{x})|\mathbf{o}(\mathbf{x})). \quad (7)$$

Equation (7) can be expressed as

$$\widehat{\mathbf{g}}(\mathbf{x})_{opt} = \sum_{\mathbf{g}(\mathbf{x})} \mathbf{g}(\mathbf{x}) \frac{p(\mathbf{g}(\mathbf{x}), \mathbf{o}(\mathbf{x}))}{p(\mathbf{o}(\mathbf{x}))} = \frac{\sum_{\mathbf{g}(\mathbf{x})} \mathbf{g}(\mathbf{x}) p(\mathbf{o}(\mathbf{x})|\mathbf{g}(\mathbf{x})) p(\mathbf{g}(\mathbf{x}))}{\sum_{\mathbf{g}(\mathbf{x})} p(\mathbf{o}(\mathbf{x})|\mathbf{g}(\mathbf{x})) p(\mathbf{g}(\mathbf{x}))}. \quad (8)$$

Only a subset of Γ is accessible in the search region of central pixel x_i. We refer to this subset as the search region, $\text{SR}(\mathbf{x})) = \{\mathbf{g}_1(x), \mathbf{g}_2(x), \mathbf{g}_3(x), ..., \mathbf{g}_K(x)\}$. Assuming that cardinality of SR is K, and $p(\mathbf{g}(\mathbf{x}))$ is uniformly distributed, i.e., $p(\mathbf{g}_i(x)) = 1/K$, for all $(0 \le i \le K)$, Equation (8) simplifies to

$$\widehat{\mathbf{g}}(x_i) = \frac{\sum_{j=1}^{K} \mathbf{g}(x_j) p(\mathbf{o}(x_i)|\mathbf{g}(x_j))}{\sum_{j=1}^{K} p(\mathbf{o}(x_i)|\mathbf{g}(x_j))}, \quad (9)$$

where $\widehat{\mathbf{g}}(x_i)$ is the optimal estimator based on the uniform distribution assumption. Given the ground truth is not accessible, we substitute the observed value of the neighbourhood patches to get

$$\widehat{\mathbf{g}}(x_i) = \frac{\sum_{j=1}^{K} \mathbf{o}(x_j) p(\mathbf{o}(x_i)|\mathbf{o}(x_j))}{\sum_{j=1}^{K} p(\mathbf{o}(x_i)|\mathbf{o}(x_j))}. \quad (10)$$

3

Given that the noise in the RF data is modelled as an additive guassian noise [22, 23], we have

$$\mathbf{o(x)} = \mathbf{g(x)} + \mathbf{v(x)}, \qquad (11)$$

where $\mathbf{v(x)}$ is the additive white guassian noise with variance σ^2. By assuming that the likelihood can be factorized as

$$p(\mathbf{o}(x_i)|\mathbf{o}(x_j)) = \prod_{k=1}^{n} p(o(x_{i,k})|o(x_{j,k})), \qquad (12)$$

where $x_{i,k} \in N(x_i)$ and $x_{j,k} \in N(x_j)$ are the counterpart pixels in the patches with central pixels x_i and x_j. Therefore, $p(\mathbf{o}(x_i)|\mathbf{o}(x_j))$ is multivariant normal distributed $p(\mathbf{o}(x_i)|\mathbf{o}(x_j)) \sim \mathcal{N}(\mathbf{o}(x_j), \sigma^2 I_n)$. Notation I_n is the identity matrix. Thus, the filter in (10) can be adapted to remove the noise of RF data as

$$\widehat{\mathbf{g}}(x_i) = \frac{1}{C(x_i)} \sum_{j=1}^{K} \exp^{-\frac{\|\mathbf{o}(x_i)-\mathbf{o}(x_j)\|^2}{h^2}} \mathbf{o}(x_j),$$

with $\qquad C(x_i) = \sum_{j=1}^{K} \exp^{-\frac{\|\mathbf{o}(x_i)-\mathbf{o}(x_j)\|^2}{h^2}}.$

$$(13)$$

Equation (13) is also known as NLM algorithm. By considering the normal distributed assumption, equation (13) can be adapted for denoising the RF data by replacing $h^2 = 2\sigma^2$. Therefore, the adapted filter for denoising the RF data (CODE) is

$$\widehat{\mathbf{g}}(x_i) = \frac{1}{C(x_i)} \sum_{j=1}^{K} \exp^{-\frac{\|\mathbf{o}(x_i)-\mathbf{o}(x_j)\|^2}{2\sigma^2}} \mathbf{o}(x_j),$$

with $\qquad C(x_i) = \sum_{j=1}^{K} \exp^{-\frac{\|\mathbf{o}(x_i)-\mathbf{o}(x_j)\|^2}{2\sigma^2}}.$

$$(14)$$

This filter is based on the noise statistics of RF data. CODE is, therefore, the optimal denoising approach for removing noise in model (11) within the RF domain.

Kevrann et al. [12] and Coupe et al. [10] have developed similar Bayesian estimators but for reducing the speckles pattern in the B-mode image. Aligned with the mathematical Bayesian estimator, the properties of noise in RF data show the usefulness of CODE for removing noise from the RF ultrasound data.

4. SIMULATION VALIDATION FOR CODE

To assess the performance of CODE approach, the Field II [24] software is used to simulate RF data from a lesion phantom of size 60, 50 and 10mm in axial, lateral and out-of-plane directions respectively. The phantoms consist of two classes of background and target tissues. To determine the precision and sensitivity of the CODE, three different setups with 5, 10, and 15 scatterers per resolution cell distributed randomly within the phantom are used. Different realizations for each group of scatterers are generated. The RF output of Field II is corrupted by adding additive white Gaussian noise with a SNR of 5dB.

Figure 3 shows the results of NLM applied to B-mode images. As expected, NLM performs incoherent averaging and removes speckle pattern. This is desired for many applications such as segmentation and registration [25], but not in elastography. Figure 3 also shows the results of applying a Gaussian kernel to the RF data. Since averaging is performed in the RF domain, the speckle pattern is retained. Finally, the results of CODE denoising is also shown in this figure, which visually outperforms other methods in terms of similarity to the original B-mode image. Figure 4 compares the histogram of the B-mode of these three images. Since the distribution of noise-free image (ground truth) is known, we used the following chi-square test as a quantitative parameter for comparison

$$\chi^2 = \sum_{t=1}^{m} \frac{(O_t - E_t)^2}{E_t}, \qquad (15)$$

where O_t is the observed value, E_t the expected value and m number of bins (256 bins of grey levels for simulated images). The Chi squared criterion for distribution and sum of squared difference (SSD) between original and filtered images using NLM, Gaussian with kernel width of 5 and smoothing parameter 1, and CODE with search region 21, kernel width 5 and smoothing parameter 5, are compared in Table 1. In both cases (Chi squared and SSD), CODE outperforms the conventional NLM approach and Gaussian denoising applied directly on RF data, as demostrated in theory in Section 3.2.

5. PHANTOM AND *IN-VIVO* ELASTOGRAPHY

Phantom data in Figure 5 is obtained from a CIRS breast phantom (Norfolk, VA) under freehand palpation. There is excessive out of plane motion between the two processed images, and therefore, the DP step fails. This leads to failure in both DPAM and GLUE, which is apparent as black horizontal artifacts in in (a) and (c). However, CODE removes the noise from the RF data and leads to a strain image with low noise and high contrast.

Patient data in Figure 6 was acquired from a patient undergoing open surgical radio-frequency thermal ablation for primary or secondary liver cancer. This data is available online [2]. The Institutional Ethical Review Board at Johns Hopkins University approved all experimental procedures involving human subjects. For the patient data, ablation procedure generates substantial amount of noise in the RF data [19, 20, 21]. As a result of excessive noise, DP fails, which generates the horizontal black and white band in the top left of (a)

Table 1. Values of Chi squared and SSD for reconstructed images. The ground truth was obtained from a Field II simulation.

Scatterers	Chi²			SSD		
	NLM	Gaussian	CODE	NLM	Gaussian	CODE
5/mm³	9702.33	167.78	**95.49**	76250.03	2240.55	**2119.90**
10/mm³	86361.68	253.65	**60.18**	7482.55	2242.51	**1909.44**
15/mm³	6108.31	294.90	**27.63**	7961.00	2300.42	**1536.20**

(a) Ground-truth   (b) Noisy   (c) NLM   (d) Gaussian on RF   (e) CODE
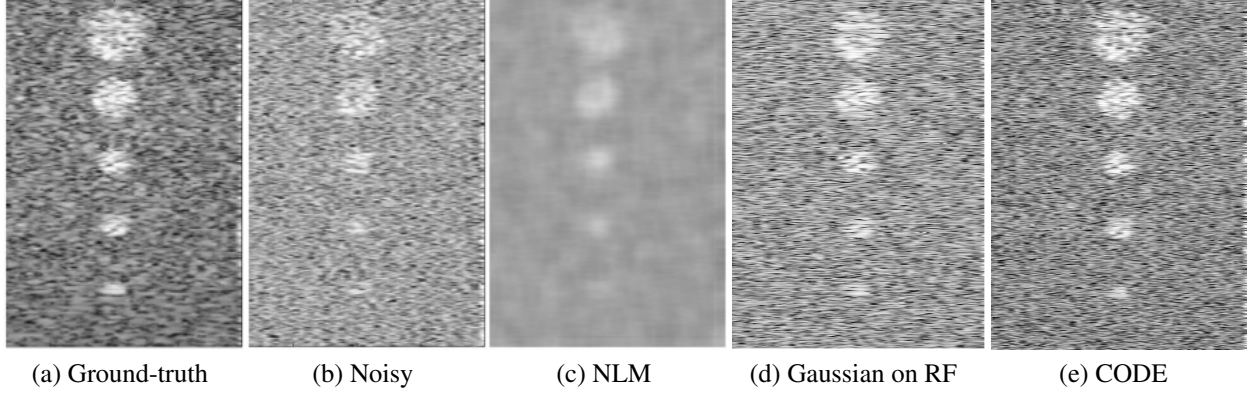
Fig. 3. Field II simulation results. The noisy input has substantially less contrast than the ground-truth image. NLM is designed to remove speckle and therefore substantially reduces image detail. CODE output is closest to the ground-truth.

Fig. 4. (a) is histograms of NLM, ground truth and noisy data. (b) is histograms of Guassian denoising, ground truth and noisy data. Finally, (c) is same as figure (b) except the histogram of NLM is replaced by that of CODE.

(a)   (b)   (c)

(a)   (b)   (c)   (d)
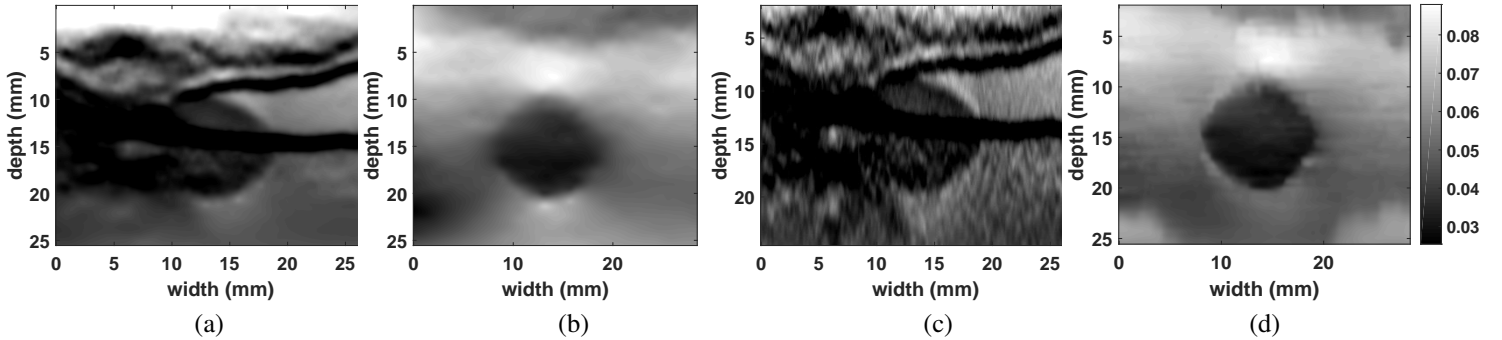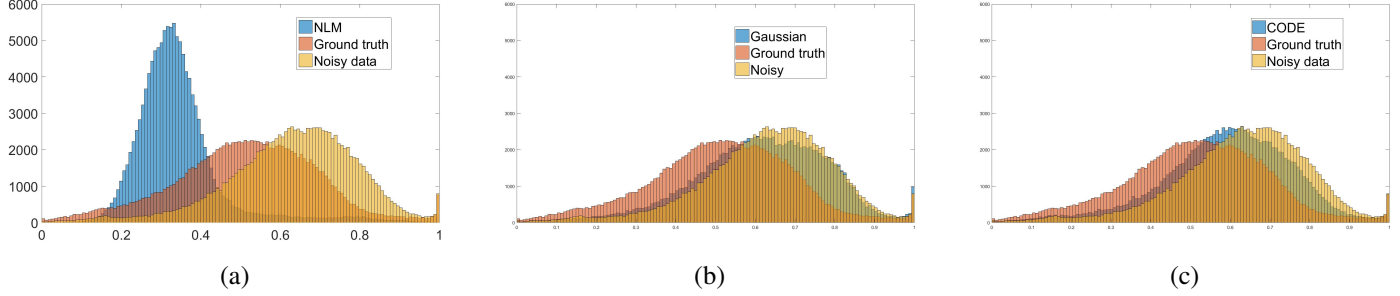
Fig. 5. Denoising results for phantom data: (a) GLUE alone; (b) GLUE with CODE; (c) DPAM alone; (d) DPAM with CODE. The dimensions of the search window is (11×11), size of the neighbourhood is (3×3), and the smoothing parameter h is set to 11.
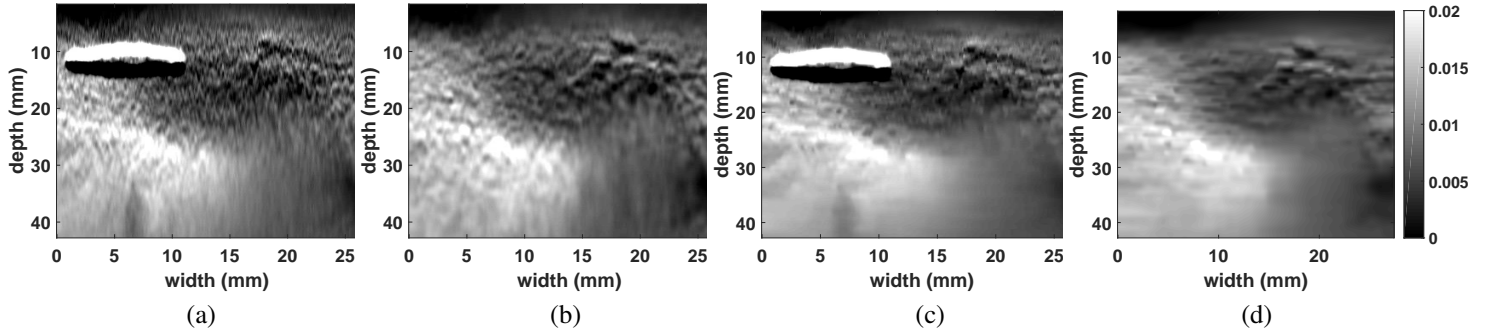
Fig. 6. Same as Fig. 5 except in-vivo liver ablation patient data is used: (a) GLUE alone; (b) GLUE with CODE; (c) DPAM alone; (d) DPAM with CODE.
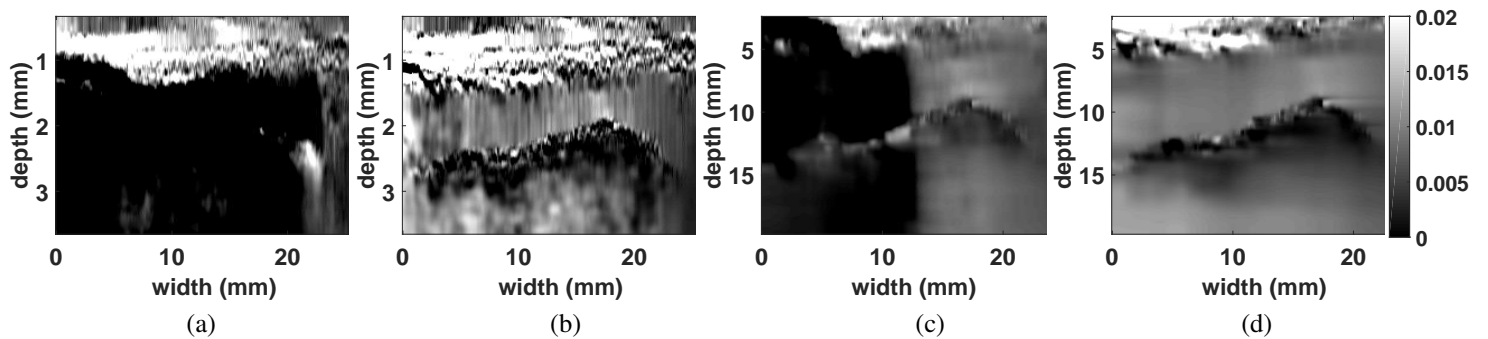
Fig. 7. Same as Fig. 5 except in-vivo patient tendon data is used: (a) GLUE alone; (b) GLUE with CODE; (c) DPAM alone; (d) DPAM with CODE.

and (c). Although the environment is extremely noisy, the well-adapted CODE method denoises the RF data in a way that both algorithms are able to get the correct strain map for patient data. The ablation operation coagulates the tissue, which makes the tissue stiffer. The coagulated tissue is often referred to as ablation lesion, and its size should be bigger than the tumor to ensure that the entire tumor is ablated. The strain images in (b) and (d) clearly show the ablation lesion as a dark region with low strain (i.e. hard). CODE helps remove noise in RF data, which leads to less noisy strain images. Such strain images can help the surgeon to minimize the cancer recurrence rate.

We also evaluate CODE on data collected from patellar tendon. This data was collected at the PERFORM Centre at Concordia University. Ethics approval was obtained for this study from Quebec's Ministere de la Sante et des Services Sociaux and all subjects signed a consent form to participate. Data is collected using an Alpinion ECube ultrasound machine (Bothell, WA) with a L3-12 linear transducer and at the center frequency of 11MHz with sampling frequency of 40MHz. The results are shown in Figure 7. The probe is held stationary, and the subject flexes his knee joint during data collection. CODE removes the noise in the RF data and results in a more meaningful strain image.

The data collected for thispublication cannot be shared online due to the requirements of the ethics approval and other constraints. Some of the data used in this paper are available online [2].

6. CONCLUSIONS

In this paper, we have proposed a denoising algorithm, referred to as the CODE (COherent Denoising for Elastography) approach for ultrasound elastography. CODE is applied directly to the RF data, and has the ability to eliminate noise, while retaining relevant speckle patterns. This is demonstrated using phantom and experiments based on *in-vivo* clinical data. The results of CODE are used for GLUE and DPAM, which verifies the effectiveness of the proposed CODE. More clinical studies are needed to fully verify the benefits of the CODE algorithm.

7. ACKNOWLEDGEMENT

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